CS451 Pipeline Performance Homework

1. In theory, what is the maximum speedup you can possibly achieve by converting the book’s single-cycle CPU into a 5-stage pipelined CPU?

Assume

•the time of the single-cycle CPU is 925,

•the pipeline stages are separated by registers that take time 50, and

•a “perfect world” where the CPU can always be divided into 5 even stages.

If we divide the 925 into 5 even segments, each stage will take 185ps, but we add the overhead of 50 ps for the additional registers. Since they can all run in parallel the cycle time should be 235ps. We can then construct a speedup formula, that relies heavily on the number of instructions.

(925 \* n) / ((k-1) + n) \* 235

(925 \* n) / ((5-1) + n) \*235 = .98

925n / (4 + n)235

925n / 940 + 235n

------------- Wolfram-----------

(185 n)/(47 (n+4))

If we exclude the miniscule number of cycles to fill the pipeline, we can interpret the speedup as such:

925n / 235n = 3.936

1 - 3.936 = 2.936

So in a perfect world, the CPU speed up for a pipeline CPU compared to a single cycle CPU is a 293.6% speed up. It is worth noting that 3.936 is also the limit of 185n/47 (n + 4).

2. In theory, what is the maximum speedup you can possibly achieve by converting the book’s single-cycle CPU into a pipelined CPU of any depth?

Assume

•the time of the single-cycle CPU is 925,

•the pipeline stages are separated by registers that take time 50, and

•a “perfect world” where the CPU can always be divided into k even stages.

The function used to model the speedup can be expressed as follows:

925 n / ((925/k) + 50) n

If we take the limit of this function as k → ∞ we get a maximum speedup of:

925/50 = 18.50

1 - 18.50 = 17.50

So theoretically we could achieve a speedup of 1750% provided we are able to segment a cpu into an infinite number of pieces.

3. In addition to the assumptions given in Problem 2, also assume that the CPI for a 5-stage pipeline is 1.035 and that the number of stalls increases by 50% with every additional stage. (In other words, assume that the CPI for a 6-stage pipeline is 1+(.035)(1.5), the CPI for a 7-stage pipeline is 1 + (.035)(1.5)(1.5), etc.)

(a) Develop a formula for the performance of this k-stage pipeline. Your formula should combine the cycle time formula from Problem 2, with the model for stalls discussed above.

f(k) = ((925/k) + 50) \* n \* (1 + ((.035) \* (1.5 ^ (k - 5)))

(b) Use your formula to determine the optimal number of pipeline stages. (The math for this problem gets complex. I suggest using gnuplot, Maple, or Wolframalpha.com to graph the solution and make a close visual guess.

f(4 - i) = undefined

f(5) = 243.225 n

f(6) = 214.885 n

f(7) = 196.486 n

f(8) = 185.189 n

f(9) = 179.949 n < - - - - optimal

f(10) = 180.373 n

After calculating the above results, we can see that a pipeline with 9 stages gives us the best performance increase when in cooperating a growing CPI.